

Calibration of a superconducting gravimeter by comparison with an absolute gravimeter FG5 in Boulder

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Abstract. This paper reports the results of a calibration of a superconducting gravimeter using simultaneous measurements of Earth tides with an absolute gravimeter. Nine consecutive days of observations were made with both the C024 superconducting gravimeter and the FG5-202 absolute gravimeter at the NOAA Table Mountain Gravity Observatory near Boulder, Colorado. The precision of the calibration factor is better than 0.1%. The calibration factor obtained in this fashion agrees well with that obtained from a moving platform. This experiment provides a noise estimate for both the superconducting and absolute gravimeter. In addition, the local air pressure admittances of both instruments compare well with a value close to $-0.35 \mu\text{Gal}/\text{mbar}$.

Introduction

Superconducting relative gravimeters (SG) and absolute gravimeters (AG) are very precise instruments. SG users typically report that it is possible to measure tidal amplitudes (semi-diurnal and diurnal) with a precision of about 1-2 nanoGal for integration periods of 2-3 years. This figure of merit can be translated into a standard noise figure of about $10 \mu\text{Gal}/\sqrt{\text{Hz}}$ independent of integration time. This noise figure is similar to that obtained from an FG5 absolute gravimeter. At a quiet site, it is possible to obtain individual gravity measurements with a precision of about $7 \mu\text{Gal}$. In the normal mode of operation the measurements are repeated every 10 seconds. Assuming that each measurement is uncorrelated, this results in a noise estimate of $22 \mu\text{Gal}/\sqrt{\text{Hz}}$ for the FG5. The best precision on a single measurement using an FG5 is about $3 \mu\text{Gal}$ [Courtier, personal communication, 1997]. Assuming data taken at the fastest rate of about 2 seconds per drop this yields a best case noise figure of about $5 \mu\text{Gal}/\sqrt{\text{Hz}}$.

The accuracy of the absolute gravimeters is also very high. The formal error estimate of the FG5 instrument is about 1-2 μGal [Niebauer *et al.*, 1995]. Intercomparisons made at the Table Mountain Gravity Observatory (TMGO) in Boulder demonstrate that the agreement between different FG5 absolute

gravimeters is at the $1 \mu\text{Gal}$ level [Klopping *et al.*, 1997]. Although it is impossible to rule out systematic errors, the absolute instruments derive their basic accuracy from a direct tie to fundamental standards of length and time using lasers and atomic clocks.

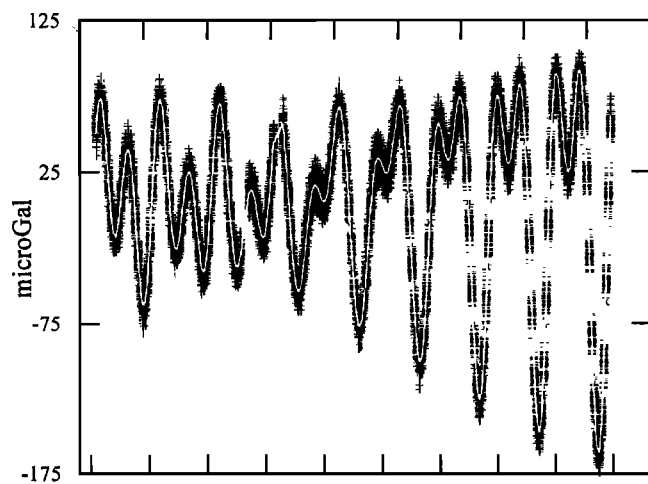
SGs balance the weight of a superconducting niobium sphere using a force produced by current in a superconducting coil. Changes in the current needed to maintain the sphere at a zero position can be interpreted as gravity changes if: (1) the mass of the sphere remains constant and (2) changes in the current are proportional to the force on the sphere. In theory, it should be possible to tie the weight measurements of the sphere to absolute standards. However, efforts to do so at standards laboratories [Robinson and Kibble, 1997; Steiner *et al.* 1997] thus far are limited at parts in 10^{-8} . Without a direct tie to absolute standards the SG is classified as a relative instrument. Changes in the superconducting coil current must be calibrated or converted to units of acceleration by external means. SGs have a very high precision and are often used to measure tidal amplitudes at the 1 nGal (10^{-12} g) level by integrating over periods of 2-3 years. At this level of precision, a number of geophysical problems can be addressed such as Earth tides and nearly diurnal free wobble, core modes, ocean and atmospheric loading, Earth rotation and polar motion, and sea level changes. These applications require a calibration with an accuracy of about 0.1 %.

There are roughly three methods for calibrating a SG. One method is to monitor the tides for a long period (2-3 years) and then use a well known tidal amplitude (e.g. O_1) to calibrate the instrument [Melchior, 1994]. This method works well, however, it is very time consuming and one is ultimately tied to uncertainties in the tidal amplitude used to calibrate the instrument. Another disadvantage of this calibration method is that unmodelled changes in air pressure, water table, or even man-made gravity can contaminate the measurement.

Another method is to accelerate the instrument using a moving platform and correlate the platform accelerations to measured weight changes in the superconducting gravimeter [Richter *et al.*, 1995]. This method is very accurate because it provides a direct tie to acceleration that can be measured externally using metrological standards of length and time. This method is more insensitive to external gravity changes because of the relatively fast platform motions and because it is possible to create large

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Figure 1. Parallel registration of the C024 superconducting gravimeter (white line) and the FG5-205 absolute gravity meter (crosses) at the NOAA Table Mountain Gravity Observatory.

accelerations. Unfortunately, phase shifts caused by the transfer function of the SG must be considered because the acceleration changes are modulated at a rate comparable to the instrument response time. One final disadvantage of this method is that the gravimeter must be temporarily installed on the platform, disturbing the observational time series.

The third approach is to use simultaneous measurements of tidal gravity changes using a calibrated gravimeter. The SG calibration is then obtained by a direct correlation of the two gravimeter outputs. The primary advantage of this method is that the SG is not disturbed and the calibration is completely independent of any unmodelled common-mode gravity changes caused by air-pressure, water-table, etc... Such a method has been already tested by *Hinderer et al.* [1991] to calibrate the SG T005 with a JILA-5 in Strasbourg. The obtained precision in the calibration factor was better than 1 percent using 24 hr continuous parallel registration. In the present experiment, we used a more accurate absolute gravimeter FG5 and 9 days of data.

Given the similarity of the background noise levels (about $10 \mu\text{Gal}/\sqrt{\text{Hz}}$) of the superconducting and absolute gravimeters, it is quite natural to use the absolute gravimeter to calibrate the

superconducting gravimeter. The calibration procedure is quite simple: the absolute gravimeter is setup next to the superconducting gravimeter and simultaneous measurements of gravity are cross-correlated to obtain the calibration factor. The necessary integration time can be calculated before the calibration begins by scaling the instrument precision at the site.

In our experiment the absolute gravimeter was run at a rate of 100 drops every hour with a single-drop precision of about $7 \mu\text{Gal}$. This corresponds to a precision of about $40 \mu\text{Gal}/\sqrt{\text{Hz}}$. Assuming a time domain amplitude of the tidal signal of about $80 \mu\text{Gal}$ (i.e. with a standard deviation of $50 \mu\text{Gal}$), and a required precision of 0.1%, the predicted integration time is $(t = (40/0.05)^2\text{s})$ 1 week. As we will see from the results of this experiment, this integration time is close to what was observed.

Calibration Experiment

The FG5-205 operated side by side with the C024 for a period of nine days from 20-28 July 1996. The absolute gravity data set (Figure 1) consists of 208 100-drop sets observed hourly at the rate of 1 drop by 10 seconds. Eight sets were rejected due to disturbances caused by earthquakes. This yields to a total of 20,800 drops with a standard deviation of single drop of $6.35 \mu\text{Gal}$. Spurious data were rejected using a boxcar window of $\pm 200 \mu\text{Gal}$.

The original SG data series is a continuous registration with one sample every 5 seconds. We used the tidal output data which are high-pass filtered using an analog filter with a cut-off period of about 70 seconds. The data contaminated by the earthquakes were discarded. We selected the SG data at times corresponding to the individual drops of the FG5. This procedure removes any contamination caused by discarded data on the cross correlation.

The absolute gravity measurements are linearly fit to the SG data. The calibration of the SG is given by the slope of the best fit line in $\mu\text{Gal}/\text{Volts}$. Data which fall outside the range of three standard deviations are rejected. 99.6% of the data remained and are used in a second fit to recalculate the SG calibration factor.

Our analysis gives a calibration factor of -80.281 ± 0.063 (or $\pm 0.08\%$). This value is very close to the conversion factor of $-80.341 \pm 0.009 \mu\text{Gal}/\text{Volt}$ found by an independent calibration using an acceleration platform built by Richter [*Robertson, personal communication, 1997*]. The correlation factor is very high and the mean square error of the residues on the fit is $6.35 \mu\text{Gal}$ corresponding to the standard deviation of the absolute

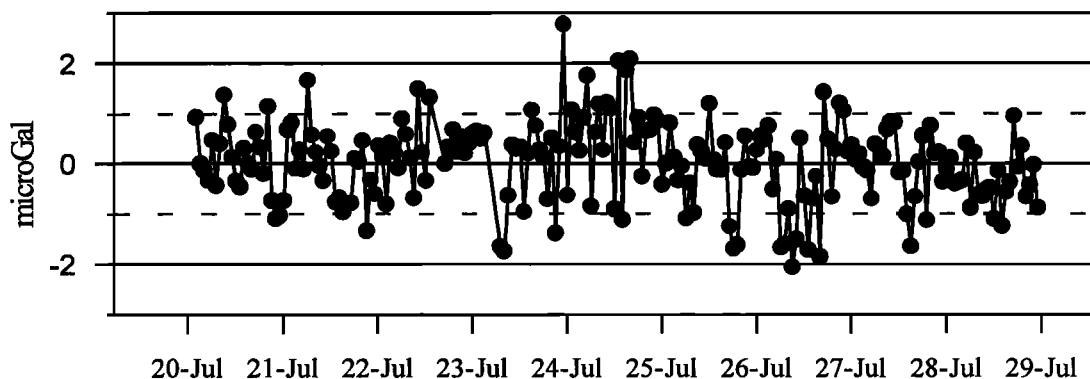


Figure 2. Residuals of the linear regression between the C024 superconducting and the FG5-205 absolute observations. Each point represents the average of 100 data from the absolute data set. The maximum and minimum differences are 2.783 and $-2.051 \mu\text{Gal}$, and the standard deviation is $\pm 0.812 \mu\text{Gal}$.

Table 1. Results of the linear regression between data of the C024 superconducting gravimeter and observations of the FG5-205 absolute gravimeter.

Parameter	Gravity FG5 / C024	Fit Residues / Atmospheric pressure
Number of data	20,713	20,713
Scale Factor	-80.281 ± 0.063 $\mu\text{Gal/Volt}$	0.023 ± 0.024 $\mu\text{Gal/mbar}$
Correlation Coefficient	-0.9936	0.0066

data. The quality of this calibration is consistent with that obtained by Francis [1997] at a different site. The new important result here is the close agreement between two different techniques of calibration.

The fit residuals are shown in Figure 2. These residuals (i.e. the differences between the calibrated superconducting data and the raw absolute data) have been low-pass filtered by computing the mean gravity values for each 100-drop set of absolute data. All the common gravity signals (tides, atmospheric pressure effects, etc.) are thus removed. The agreement between both instruments is excellent. The standard deviation of the 1-hour averaged data is $0.81 \mu\text{Gal}$ with a maximum difference peak-to-peak of $4.8 \mu\text{Gal}$. This precision is somewhat better than our noise estimate of about $40 \mu\text{Gal}/\sqrt{\text{Hz}}$. A similar conclusion has been obtained by Okubo *et al.* [1997].

To verify that common mode gravity signal does indeed cancel out in this process, we compare the fit residuals to an atmospheric pressure time series. The atmospheric pressure was recorded simultaneously with the gravity data. The adjustment of the fit residuals to the atmospheric pressure data gives a slope close to 0 and very low correlation coefficient (Table 1). There is thus no significant difference in the sensitivity of both instruments to the barometric effect.

Atmospheric pressure admittance

In the previous section, we demonstrated that the C024 and the FG5 meters measure the same gravity variations. We can test

the precision of both meters by attempting to extract a common gravity signal from each time series. The largest common signal is the Earth and ocean loading tides. However, with only 9 days of data, it is not possible to separate the main tidal components. The next largest common mode gravity signal is atmospheric pressure. Atmospheric pressure variations produce changes in gravity of -0.3 to $-0.4 \mu\text{Gal/mbar}$.

The tidal signal was removed from the raw data of the SG and the FG5 by subtracting a synthetic tide based on the tidal parameters derived from the analysis of 2 years of the C024 data [van Dam and Francis, 1998]. The tidal residuals of the SG and the FG5 were both low-pass filtered by averaging the data within each 100-drop set. The averaged residuals were then fit to the local atmospheric pressure data. The admittance factors (Table 2) are $-0.34 \pm 0.19 \mu\text{Gal/mbar}$ and $-0.36 \pm 0.09 \mu\text{Gal/mbar}$ for the FG5 and the SG, respectively. The admittances are in close agreement and compare well with the admittance $-0.356 \pm 0.001 \mu\text{Gal/mbar}$ deduced from 2 years of SG data [van Dam and Francis, 1998]. The large error bars on both determinations are due to the short duration of the data set (only 9 days). The uncertainty on the FG5 determination is about twice the one of the SG and the correlation coefficient is smaller. Similar uncertainties could be obtained if the data sampling of the FG5 is increased to 400 data/hour.

This data set is too short for an extensive analysis of the influence of barometric pressure variations on gravity [see, for example, Warburton and Goodkind, 1977]. However, a detailed discussion on the atmospheric pressure admittance at TMGO can be found in van Dam and Francis [1998].

Discussion

The precision of the calibration factor on the SG using an FG5 depends on the duration of the simultaneous registration. The question becomes "how many days of observations do we need to obtain a precision of 0.1% required for geophysical investigations?". To address this issue, we determined the convergence rate of the scale factor as a function of the length of the data series. The data set was truncated to varying daily lengths and the corresponding scale factor calculated. We then computed the percentage difference with respect to the scale

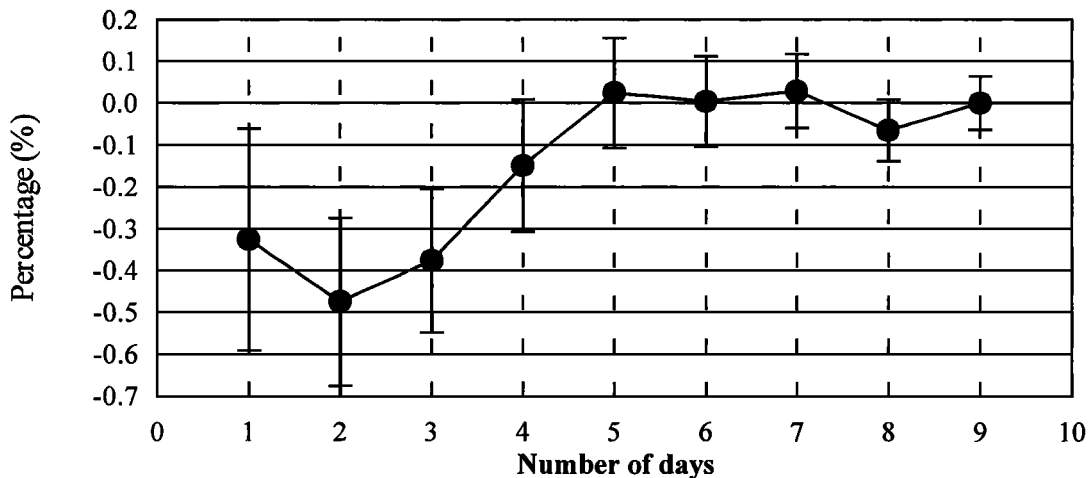


Figure 3. Percentage difference between the value of the scale factor as function of the length of the data series and the reference scale factor. The reference scale factor is defined as the scale factor value estimated by using the nine days of data.

Table 2. Air pressure admittance determined by least square adjustment of the local air pressure data to the tide free gravity data.

Parameter	C024	C024	FG5-205
	2 Years of data	20-28 Jul 1996	20-28 Jul 1996
Admittance factor ($\mu\text{Gal}/\text{mbar}$)	-0.3559	-0.36	-0.34
Standard deviation ($\mu\text{Gal}/\text{mbar}$)	0.0008	0.09	0.19
Correlation coefficient		-0.94	-0.57

factor determined using the whole data set of nine days (Figure 3). After 5 days, a convergence was obtained and the scale factor remained stable at 0.05%. The convergence is determined by the signal to noise ratio. The noise factor is a site dependent value that decreases as the square root of the observation time. The signal power can be estimated by the variance of the data (because the time varying signals are much larger than the noise). At short time intervals (less than one tidal period) the tidal signal depends on the initial phase of the tides when the calibration is started. Thus a faster convergence is obtained by starting the observation period when the tidal signal is maximum.

The fact that the calibration precision of 0.1% was obtained in about 5 days is in agreement with the $40 \mu\text{Gal}/\sqrt{\text{Hz}}$ noise estimate for the FG5 derived from the precision of a single measurement. This is good evidence that the precision of the absolute gravimeter scales as the square root of observation time.

In this study, we addressed the problem of SG "amplitude" calibration factor. In addition to the amplitude calibration, the phase of the calibration factor needs also to be experimentally determined (at a precision of 0.02 degree to meet the scientific objectives [Hinderer et al., 1991]). The phase calibration cannot be obtained with the method described in this paper. The most accurate method consists of adding a known sine wave or a step function voltage into the feedback loop of the superconducting gravimeter [Wenzel, 1994]. The frequency transfer function is then deduced by comparison of the input and output signals.

Conclusion

We have demonstrated that the calibration factor of a superconducting gravimeter (SG) can be estimated with a precision of 0.1% by operating the SG and an absolute gravimeter FG5 simultaneously for at least 5 days. The results are in close agreement with a independent calibration method using an accelerating platform. The estimates of the atmospheric admittance factor for the SG and the FG5 at Boulder are in close agreement with a value close to $-0.35 \mu\text{Gal}/\text{mbar}$. The comparison between the SG and the FG5 data is in agreement with a noise estimate of about $40 \mu\text{Gal}/\sqrt{\text{Hz}}$ for the FG5 where the precision decreases as the square-root of the observation time.

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